An Effective PML for the Absorption of Evanescent Waves in Waveguides

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Abstract—**As emphasized by several authors in literature, the evanescent modes are not absorbed by usual perfectly matched layers (PML's) terminating waveguiding structures. The purpose of this letter is to present a new version of the PML that allows a substantial absorption of such waves to be achieved.**

Index Terms—**Absorbing boundary conditions, FDTD method, waveguide.**

I. Introduction

The absorption of evanescent waves at the end of waveguides terminated by perfectly matched layers (PML's) has been addressed in several papers [1]–[3]. Although evanescent waves in general are absorbed by PML's [4], in such problems they are not absorbed, due to the fact that the direction of evanescence is perpendicular to the vacuum–PML interface, i.e., the direction of propagation is parallel to the interface. The result is that the reflection is at best equal to the natural decrease in the PML thickness [1]. In this letter we show that the modified PML defined in [5], denoted as PML-D, allows a strong absorption of evanescent waves to be achieved, resulting in an overall reflection far smaller than the reflection corresponding to the natural decrease.

II. The PML-D

The PML-D medium is obtained by means of splitting the usual PML. Each subcomponent is split into two parts so that different conductivities denoted by σ_a and σ_b can be assigned to each part. For example, H_{zx} is split into H_{zxa} and H_{zxb}. In three dimensions (3-D), there are 24 subcomponents in the PML-D. The two-dimensional (2-D) equations are given in [5]. The extension to 3-D is straightforward. The parameters that define the medium (see [5]) are the splitting parameters p_a and p_b (with $p_a + p_b = 1$), and the ratio of the profiles of conductivity σ_a and σ_b. This ratio $s = \sigma_a/\sigma_b$ must be set to a large value ($s > 10$) to obtain interesting properties (if $s = 1$, PML-D reduces to PML).

In theory, the properties of PML-D are quite close to those of PML: no reflection from interfaces, no absorption of waves whose propagation is parallel to the interface. In practice, this medium reduces the numerical reflection in interaction problems [5]. We will see below that PML-D can also act as a numerical absorber of evanescent waves whose evanescence is perpendicular to the PML.

Manuscript received December 2, 1997.

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Publisher Item Identifier S 1051-8207(98)03451-5.

For a general nonhomogeneous wave characterized by its evanescence coefficient $\cosh\chi = (1+\sinh^2\chi)^{1/2}$, traveling in a direction forming an angle φ with the normal to a PML-D having only longitudinal conductivities (side PML-D), the theory of numerical reflection yields the following reflection coefficient R for a N-cell thick PML-D of conductivity profiles σ_a and σ_b:

$$M \begin{bmatrix} R \\ T(1/2) \\ T(1) \\ \vdots \\ T(N-1/2) \end{bmatrix} = \begin{bmatrix} V \\ \alpha\, C^*(1/2) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{1}$$

where M is the tridiagonal matrix

$$M = \begin{bmatrix} U & \alpha C(0) & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & -\alpha C(L) & 1 & \alpha C(L) & \cdots \\ \cdots & -\alpha C^*\left(L+\frac{1}{2}\right) & 1 & \alpha C^*\left(L+\frac{1}{2}\right) & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & -\alpha C^*\left(N-\frac{1}{2}\right) & 1 \end{bmatrix}$$

and

$$\alpha = \frac{c\Delta t}{\Delta x}\,\frac{1}{\phi}, \qquad \phi = \cosh\chi\,\cos\varphi - j\,\sinh\chi\,\sin\varphi$$

$$U = 1 + \alpha\, C(0)\left[\sqrt{1-S^2} - jS\right], \qquad S = \frac{1}{\alpha}\,\sin\,\frac{\omega\Delta t}{2}$$

$$V = -1 + \alpha\, C(0)\left[\sqrt{1-S^2} + jS\right]$$

$$C(L) = \frac{p_a B_a(L)}{e^{j\omega(\Delta t/2)} - A_a(L)e^{-j\omega(\Delta t/2)}} + \frac{p_b B_b(L)}{e^{j\omega(\Delta t/2)} - A_b(L)e^{-j\omega(\Delta t/2)}}$$

$$A_u(L) = e^{[-\sigma_u(L)\Delta t/\varepsilon_0]}, \qquad B_u(L) = \frac{[1-A_u(L)]\varepsilon_0}{\sigma_u(L)\Delta t}.$$

System (1) of $2N$ equations and $2N$ unknowns (R and T quantities) can be solved recursively for R. L is the index of the mesh ($L = 0$ in the interface). Δx and Δt are the space and time steps. Quantities C^* are as C with σ^* and μ_o in place of σ and ε_o. A and B are for an exponential finite-difference time-domain (FDTD) differencing. In waveguide calculations, $\varphi = \pi/2$ (propagation parallel to the PML-D) and $\cosh\chi$ is given by the mode that is considered.

III. Numerical Experiments

We considered the TM_1 mode of the 2-D parallel-plate in [3] (40 mm thick, $\Delta x = 1$ mm, cutoff 3.75 GHz). Fig. 1

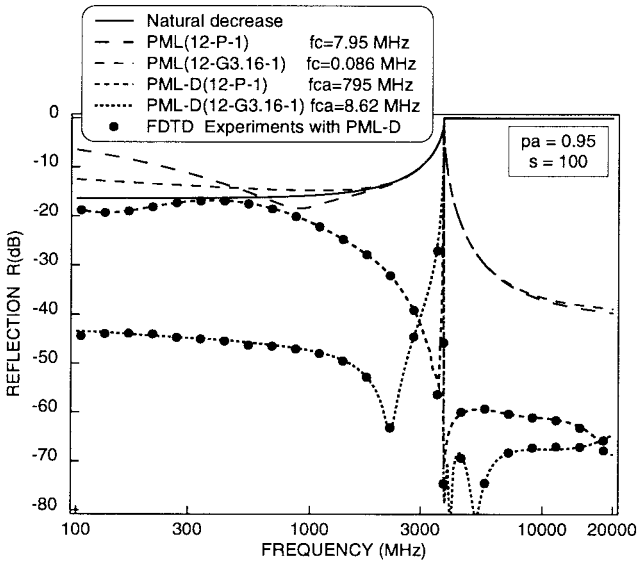

Fig. 1. PML/PML-D comparison.

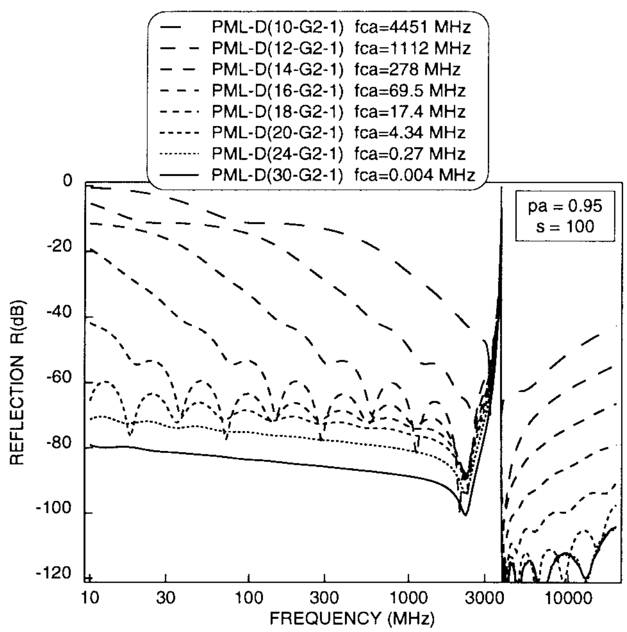

Fig. 2. Effect of the frequency f_{ca}.

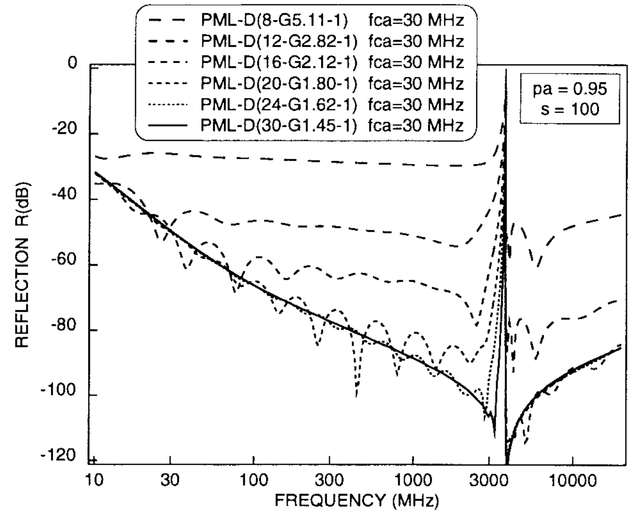

Fig. 3. Effect of the geometrical progression g.

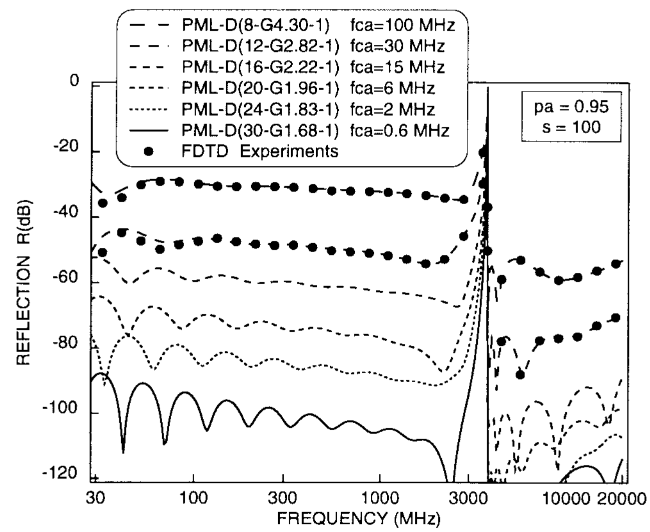

Fig. 4. Optimized PML-D's in a two-decade band of frequencies.

shows reflection R computed by (1) for 12-cell PML's and PML-D's having either parabolic or geometrical profiles of conductivity. The PML's are denoted as in [6]. The theoretical reflection $R(0)$ of every PML equals 1%. With PML-D's, $R(0)$ is calculated as with PML's [6], but using σ_b instead of σ. Fig. 1 also shows FDTD results validating R from (1).

Contrary to PML's, PML-D's allow R to be far smaller than the natural decrease of evanescent waves, especially with the geometrical conductivity. PML-D also widely reduces the reflection of traveling waves. Only a narrow band of frequencies is reflected around the waveguide cutoff. The difference between parabolic and geometrical conductivities can be explained by considering frequencies f_c and f_{ca} in Fig. 1. As known [4], the strongly evanescent waves are reflected by PML's below cutoff frequency $f_c = \sigma(0)/2\pi\varepsilon_o$ where $\sigma(0)$ is the conductivity in the vacuum–PML interface (notice that f_c is a PML parameter, not the waveguide cutoff). Similarly, with PML-D the reflection is strong below $f_{ca} = \sigma_a(0)/2\pi\varepsilon_o$. Using geometrical profiles instead of parabolic profiles, f_c and f_{ca} are far smaller, resulting in a small reflection up to far lower frequencies. As in interaction problems [6], the geometrical conductivity is the best profile when dealing with evanescent waves with PML or PML-D.

Fig. 2 illustrates the effect of parameter f_{ca}. By increasing the PML-D thickness, the frequency f_{ca} is reduced, so that the numerical reflection is shifted toward lower frequencies. Fig. 3 shows that the reflection from PML-D inner interfaces can be reduced above f_{ca} by reducing the ratio g. This figure also shows that the reflection from the vacuum–PML interface decreases approximately as $1/f$ above f_{ca}, as can be predicted by theory. In consequence, to achieve a given reflection R in applications, f_{ca} will have to be set sufficiently far from the lowest frequency of interest.

With the intention of achieving a desired R in a given band of frequencies, by using (1) the key parameters f_{ca} and g can be optimized so that the PML-D thickness is minimum. This is illustrated by Fig. 4 that shows the PML-D's giving R in the range -30 to -100 dB in a two-decade band of frequencies below the waveguide cutoff. Comparing to the respective natural decreases ranging from -10 to -40 dB, the additional numerical absorption due to the PML-D's are in the range -20 to -60 dB.

IV. Conclusion

The split PML-D [5] allows an important absorption of evanescent waves to be achieved in PML's terminating waveguides. Except for a narrow band of frequencies near the waveguide cutoff, the reflection coefficient can be reduced by several tens of decibels by using PML-D in place of PML. The computational cost of PML-D is not prohibitive, since in 3-D only 14 subcomponents must be computed in side PML-D's, due to the absence of transverse conductivity. Actually, in terms of ratio cost/efficiency, PML-D is probably far better than the normal PML in all the problems involving nonhomogeneous waves whose direction of propagation is parallel, or almost parallel, to the vacuum–PML interface.

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